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# Statistical smoothing methods: some practical aspects

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## SECOND PERIODIC REPORT

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## EQUIPMENT

During this period a second SUN workstation has been added to the resources of the statistics group. This configuration of workstations has significantly increased the computer power and graphical facilities available. This increase in computer resources has proved most useful in the computationally demanding work currently being carried out on the evaluation of image reconstruction techniques.

## SCIENTIFIC WORK

### Density estimation

The paper [1] referred to in the First Periodic Report has now been accepted for publication. This paper deals with the question of how much effect smoothing will have when using the bootstrap. One of the novel features of our investigation was the use of Computer Algebra to solve this statistical problem, and it is expected that a second publication will eventually result concentrating on this particular aspect.

### Edge process models for image reconstruction

One of the important components of the Geman approach to image reconstruction is the specification of an "edge process" whereby a stochastic process is used to model the edges present in the picture. Pixels separated by edges which are actually present are no longer considered as neighbouring in the subsequent reconstruction. Work has been continuing on the consequences of considering the pixel edge process as a discretized version of a "true" underlying edge process, and a paper describing the results obtained is now in draft. An early version [2] of this paper was published by Stanford University as a Technical Report. Considerable additional work has now been carried out on the appropriate way of dealing with "loose ends" and "branches" in the pattern, and additionally to provide practical illustration of the way that discretised versions of real images actually give rise to relatively invariant costs when the underlying picture is moved around relative to the pixel grid.
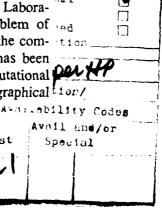
### ICM and Annealing

A large scale simulation study of the ICM approach to image reconstruction has now been carried out, making use of the programs referred to in the first periodic report. The results of this work are being prepared for publication. Work is also progressing in a number of other areas including the generalisation of ICM to non-homogeneous images.

### Nonparametric discriminant analysis

The CART approach to nonparametric discriminant analysis has now been implemented, incorporating some novel computational features to economise on computer time. The algorithm has produced encouraging results on trial data sets and the investigators are now ready to apply it to data supplied by the US Night Vision Laboratories. Current theoretical investigations are focused on the important problem of estimating misclassification rates in CART. At present work is progressing on the comparison of various plausible techniques. A considerable amount of software has been developed to grow decision trees for quantative features. Current computational developments include the representation of decision trees using the powerful graphical facilities at our disposal.

### Image Refinement

The image refinement method in which each pixel is subdivided into four subpixels, each of which can contain a different colour in the reconstruction, has been extended to allow a single straight line edge within each pixel. The Markov random field model on the usual pixel grid or on a refined grid is extended to this limiting case by a term in the exponent of the Gibbs distribution equal to a multiple of the total edge length in the image. This allows a much more realistic representation of the boundaries of regions and objects in the restored image. An algorithm to implement this method has been programmed and impressive results obtained. A paper [3] has been prepared for submission to an international conference. There are also strong links between the theoretical motivation of this work and the work on edge process models reported earlier and we are continuing to explore this connection.

### Indirectly observed images

Progress continues to be made on the two closely related problems involving the use of the EM algorithm in the reconstruction of indirectly observed images, as described in section 3 of the original proposal. The key idea is to introduce smoothing, via a simple sort of spatial averaging, at each EM iteration. Very good results have been a obtained in the one-dimensional case where the algorithm converges much more quickly than in the unsmoothed case and gives a better result which is free of unwanted rapid local variation. A paper describing the application of this "smoothed EM" procedure to a particular problem in stereology is now in preparation.

In the spatial case similar ideas are being pursued with promising results. Particular attention is being devoted to the Positron Emission Tomography problem encountered in medical imaging, but the general approach being developed will be applicable to problems of indirect imaging generally. Several different smoothing strategies have been tested, some with spectacularly good results, and a paper describing the progress made is now in draft.

One important theoretical question concerning indirect imaging is the quantification of the information loss in taking the observations indirectly rather than observing them directly. Jointly with I.M. Johnstone of Stanford, B.W. Silverman has made considerable progress in combining ideas from density estimation with properties of the eigenvalues of the linear operators involved to yield a measure of the inefficiency inherent in the indirect observation procedure.

### PUBLICATIONS

[1] Silverman, B. W. and Young, G. A. The bootstrap: to smooth or not to smooth? Accepted by *Biometrika*.

[2] Brown, T. C. and Silverman, B. W. Edge process models for regular and irregular pixels. Technical Report No. 267, Department of Statistics, Stanford University, Stanford, California.

[3] Jennison, C. and Jubb, M. Statistical image restoration and refinement. Submitted to Tenth International Conference on Information Processing in Medical Imaging, to be held in June 1987 in Utrecht, The Netherlands.

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